THE STATE OF THE ART
ON STRUCTURAL EQUATION MODELING

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Structural equation modeling (SEM) refers to a set of multivariate statistical techniques used to measure latent (unobserved) variables, which have been used more and more frequently. The study sought to analyze the current state of SEM disclosure, through technical articles. To this end, a review was carried out using the PRISMA method and data were collected in databases such as Scielo, APA PsycInfo, and Pepsic, with the descriptors SEM, Psychometrics, and review reports. Initially, 337 articles were found. However, only 16 were eligible for this research. It was found that the SEM promotes the expansion of multiple linear regression and confirmatory factorial analysis, combining these analyses. It was concluded that SEM is a robust and reliable statistical method that should be widely applied in health research, such as psychology, as it allows the technical analysis of research data.

Keywords: Structural equation modeling; Psychometrics; Systematic review; Prism methodology.

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Psychometrics is a branch of psychology dedicated to the constitution of mechanisms for measuring subjectivity and their application (Melhado & Rabot, 2021), with origins at the end of the nineteenth century related to the psychophysics of Ernst Heinrich Weber and Gustav Fechner, German psychologists. The Englishman Francis Galton contributed to the construction of tests to measure mental processes. However, it was the creator of multiple factor analysis, Leon Louis Thurstone, who gave shape to psychometrics, distancing it from psychophysics (Melhado & Rabot, 2021).

Psychometrics has evolved and transformed since Galton, around 1880, with Cattell in 1890, in 1900 with Binet, with intelligence tests between 1910 and 1930, the decade of factor analysis in the 1930s, the era of systematization in 1940 until reaching modern psychometrics today, also called Item Response Theory or IRT. This evolution has enabled the emergence of new methods, strategies, and perspectives, boosting studies in the area and promoting new fields of investigation (Pasquali, 2017).

Pasquali (2017) considers that as a science, psychometrics performs psychological evaluations without the function of measuring mental processes. To this end, instruments are built and applied to measure psychological constructs and variables, which, together with statistical analysis methods, make it possible to analyze and measure psychological states, based on metrics and knowledge from the field of psychology.

Psychometry explains the meaning of the subjects' responses. To do so, it depends on statistical models that demonstrate existing relationships between observable behaviors and psychological constructs. According to Maia and Lima (2021), the main characteristic of psychometrics is the ability to accurately express observed data. Thus, through this field of study, a dialogue is established between the theory and the technique of measuring mental processes.

Psychometrics allowed several advanced statistical techniques to be developed for data analysis. Among these techniques are: factor analysis, correlation analysis, regression analysis, path analysis, and structural equation models (Appelbaum et al., 2018).

Regarding factor analysis, the factors refer to the set of observed variables, which are related to each other. In this sense, it determines the quantity and nature of factors (or latent variables) that can explain the variation and covariation of the observed variables (e.g., items of a measurement instrument; Rogers, 2022). Factor analysis can be exploratory and confirmatory. Exploratory factor analysis or EFA is a widely used statistical technique for measurement. This technique of interdependence allows us to determine the quantity and nature of latent variables or factors that can explain variation and covariation, in a set of observed measures. It validates psychological tests as a statistical procedure (Rogers, 2022).

On the other hand, confirmatory factor analysis, or CFA, statistically tests the validity of a given previous theoretical framework on a set of observed variables. In CFA, a confirmatory analysis technique, the researcher predetermines which structure should be validated. It is a type of structural equation modeling that performs measurement. Therefore, it comprises models aimed at evaluating the relationship between observed variables and latent variables (Goretzko et al., 2024). For CFA, a ready-made factorial structure is needed, a strong conceptual basis with a clear definition of the number of factors and items that correspond to the factors, that is, the researcher specifies the number of factors, the structure and the relationship of these factors with the indicators. This conceptual basis guides the specification and evaluation of the model. It is a recommended model for later phases of school development and construct validation (Rogers, 2022).

The correlation represents the relationship between two variables, and can vary from –1.0 to +1.0. Thus, it is important that the interpretation of the correlation coefficients, statistical or theoretical, be carried out according to the behavior of the variables analyzed. It is necessary to proceed with the verification of assumptions for use as one of the means to ensure a coherent relationship between the variables (Maia & Lima, 2021).

Linear regression is a statistical measure of estimating the coefficients of a linear equation that predicts what the behavior of the dependent variable will be, based on one variable or multiple independent variables (Capp & Nienov, 2020). In other words, it performs data analysis by predicting the behavior or influence of one variable in relation to the others. Thus, the variable is explained as a function of the linear combination of the other variables (Azzari & Pelissari, 2020).

Also for the authors, path analysis is a type of linear regression. This statistical technique allows the decomposition of the correlation between two variables into other components associated with varied paths that make it possible to link these two variables in the path of the set. The main difference to linear regression is that path analysis allows the use of more than one dependent variable (Maia & Lima, 2021).

Finally, structural equation modeling (SEM) is a statistical technique whose characteristics are: (i) measurement errors associated with the variables under study are explicitly considered in the model; (ii) the ability to accommodate multiple interrelated dependency relationships of variables in a single model. Applied only from the 1970s onwards, it had its first studies at the beginning of the twentieth century after Spearman (1904) dealt with factor analysis and Wrigth (1921, 1934) with path analysis (Maia & Lima, 2021).

SEM gained consistency and became more applied after the development of the first LISREL or Linear Structural Relations software. Since then, new software has been developed making it easier to use. Between the years 2000 and 2020, about 554 studies were developed on structural equation modeling in Brazil (Maia & Lima, 2022).

SEM can be conceptualized as a set of multivariate statistical techniques for measuring latent (unobserved) variables with sets of observed indicators and then to analyze the structural relationships between latent variables or between observed covariates and latent variables (Gomide et al., 2021). It can analyze whether a model has been correctly proposed, when explaining the behavior of the data, in a confirmatory perspective context. The researcher defines, based on his knowledge, how the variables of the processes are related to explain the behavior of the data under study (Beuren & Oro, 2014; Maia & Lima, 2022).

Considering its robustness, the SEM can be used in practically all fields of study, as it allows (1) multiple relationships simultaneously, as well as statistical efficiency, and because of (2) its skillful ability to evaluate relationships between variables. Thus, it ensures greater credibility to the results (Campana et al., 2009).

Psychology allows methods for data analysis. Multivariate techniques of data analysis allow the study of complex phenomena, contributing to the advancement of this science (Pilati & Laros, 2007). The use of multivariate statistical analyses favors the theoretical and methodological development of psychology, with the testing of dozens of multiple relationships simultaneously (Pilati & Laros, 2007). Thus, statistical effectiveness is possible, as measurement ensures validity and reliability of the data (Clark & Watson, 2019).

Thus, the SEM proves to be an efficient technique for statistical analyses, as it allows the confirmatory test of a psychometric structure of measurement scales and allows the analysis of explanatory relationships of multiple variables (observed or latent) simultaneously (Pilati & Laros, 2007). For this reason, the SEM has been used for research in Organizational and Work Psychology (Pilati & Abbad, 2005), Social Psychology (Gouveia et al., 2001), and in other areas of the social sciences and humanities for the evaluation of multiple variables, simultaneous direct and indirect relationships (Pilati & Laros, 2007).

SEM has advanced and is therefore being applied in more complex analyses. Currently, it enables the selection of variables, intensive longitudinal data modeling, and the analysis of complex experiments online (Hounkpatin et al., 2017). Thus, it is used in invariance tests, exploring the equivalence of measurements of a construct between groups (Sterner & Goretzko, 2023), dynamic fit indices, the least absolute deviation (LAD) estimator for the estimation of the smallest absolute deviation (van Kesteren & Oberski, 2021), and the diagonally weighted least square (DWLS) estimator, all of which are advanced multivariate statistical methodologies.

However, SEM is not widespread compared to regression and correlation, even though it allows the simultaneous modeling of relationships of multiple independent and dependent constructs and tests explanatory relationships between multiple variables simultaneously (Dutra et al., 2016). Many technical studies address correlation (e.g., Azzari & Pelissari, 2020; Silva-Costa et al., 2019) and linear regression (e.g., Maia & Lima, 2021). However, few studies explain SEM and its applications. Considering its importance and robustness, the present study aims to expand knowledge about SEM. Specifically, we sought to analyze the current state of the dissemination of the SEM, through technical articles.

METHODOLOGY

The present study conducted a bibliographic search on the current state of the use of SEM. After the bibliography search, the methodological procedure of systematic review was adopted, a method of great importance to allow an interpretative description and a more improved view of the subject under discussion (Page et al., 2021).

The systematic review was based on clear questions, a systematized and explicit method in order to promote the identification, selection, and careful evaluation of relevant studies on the subject. To this end, the PRISMA guidelines were used as a method to support the review process, enabling the identification, selection, evaluation, and synthesis of studies with more advanced methods. The PRISMA statement contains a 27-item checklist, with detailed recommendations for reporting each item (Page et al., 2021).

PRISMA has fundamental items for a clear report in a systematic review. In addition to this checklist, there is a flowchart that allows, in the results section of the systematic review, to include the total number of references found, which were excluded in each phase, and which were kept for research (Donato & Donato, 2019). Based on this method, searches were carried out in platforms and databases such as Scielo, APA PsycInfo, and Pepsic. The descriptors DECs and keywords were used for the retrieval of studies relevant to the present study. The descriptors used were: structural equation modeling, psychometrics, review reports.

The inclusion criteria were original articles in which the SEM was addressed with a focus on expanding its use. The selected manuscripts were produced and published in Portuguese and English. Technical articles that explore the SEM technique, published between the years 2015 and 2025, were selected. Nontechnical articles, literature reviews, dissertations, book chapters, and editorials were excluded, as well as manuscripts that did not deal with the SEM technique and were not produced in English or Portuguese. Repeat studies were excluded.

In this search, 337 articles were found. technical and nontechnical articles. Of these: 176 obtained from the Scielo database, 133 from the APA PsycInfo database, and 28 from Pepsic. A total of 27 duplicate articles were found, 12 in the Scielo database, 13 in the APA, and 2 in Pepsic. In addition, 253 nontechnical articles and 41 that did not fit the inclusion criteria after reading in full were excluded.

It was found that 16 were eligible for the research. These articles were selected using the descriptors and defined free terms. To this end, their identification was carried out in three stages: i) reading of titles of studies found and, consequently, exclusion of titles that did not fit the inclusion criteria; ii) reading of the abstracts of the titles selected in the previous stage. Subsequently, abstracts that did not meet the criteria selected for inclusion were excluded; iii) reading the studies in full, and then selecting the studies that met the inclusion criteria was carried out. An analysis of the data from each selected article was carried out based on a protocol file, in which aspects such as author, year, location, population/sample, objective of the studies, and main results were observed. These data were presented as a function of the relevant data of each article, in tables and figures, in order to enable an easier observation to understand the results and discussion.

RESULTS AND DISCUSSION

Figure 1 shows the flowchart of the selected articles. As can be seen, 16 articles were selected to compose the present study. Table 1 shows the information on the title, authorship and main results of the selected articles.

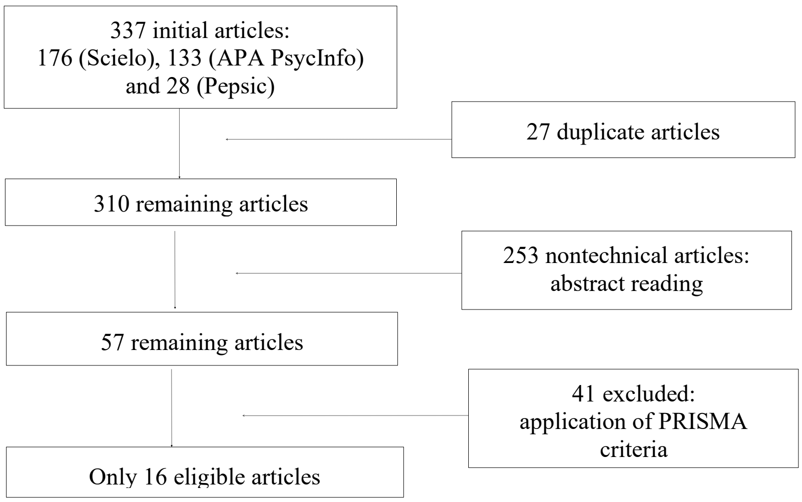

FIGURE 1
Article selection flow diagram

TABLE 1
Synthesis of the selected articles

Title	Authors	Main findings
Factor score path analysis: An alternative for SEM?	Devlieger & Rosseel, 2017	A combination of Croon's (2002) method with path analysis was performed, resulting in factor score path analysis. This method results in correct path coefficients and has some advantages over SEM: it requires smaller sample sizes, it can handle more complex models, and the method is less sensitive to misspecifications due to its stepwise nature. In conclusion, this method can be a suitable alternative to SEM when dealing with a complex model and small sample sizes
An evaluation of non-iterative estimators in confirmatory factor analysis	Dhaene & Rosseel, 2023	Closed-form expressions can serve as viable alternatives for maximum likelihood (ML), with the multiple group method — the oldest method under consideration — showing favorable results in all settings
Testing model fit in path models with dependent errors given non-normality, nonlinearity and hierarchical data	Douma & Shipley, 2022	The use of a generic method for testing path models that include dependent errors, nonlinear functional relationships, and nonnormal and hierarchically structured data. This method produces results identical to classical covariance-based path modeling when meeting its assumptions of linearity and normality, surpasses classical SEM given the nonlinear functional relationships, and can easily accommodate any parametric probability function and nonlinear functional relationship
Tutorial: The practical application of longitudinal structural equation mediation models in clinical trials	Goldsmith et al., 2018	It was described how to fit several longitudinal mediation models, including simplex, latent growth models, and latent change models. This will allow readers to learn about a model of interest or various alternative models, so they can adopt this sensitivity approach
Evaluating model fit of measurement models in confirmatory factor analysis	Goretzko et al., 2024	The model fit in many studies should be viewed critically, especially regarding the restrictions of independent clusters that are commonly imposed. Furthermore, many studies do not fully report all the results necessary to reassess the model fit
Multivariate data analysis	Hair et al., 2017	Advanced statistical techniques, including factor analysis, multiple regression, discriminant analysis, cluster analysis, SEM, and PLS-SEM, with both theoretical and practical perspectives. It provides guidelines for interpreting results and applying them across various fields, making it essential for researchers and professionals in multivariate statistical analysis

(table 1 continues)

Table 1 (continued)

Title	Authors	Main findings
A note on evaluating the moderated mediation effect	Jacky et al., 2022	To discard spurious results from a moderated mediation analysis, two methods were proposed that are simple and easy to implement. Based on the simulation results, some practical guidelines were offered to apply the methods in empirical research
Micro-macro and macro-micro effect estimation in small scale latent variable models with Croon's method	Kelcey et al., 2020	In this study, extensions were outlined for the Croon-based estimator recently developed for multilevel structural equation models. The performance of the Croon approach was then evaluated under a maximum estimator. The results suggest that the Croon method often outperforms maximum likelihood in terms of convergence, bias, and mean squared error, and represents a useful complementary estimator
Conditional process analysis for two-instance repeated-measures designs	Montoya, 2024	A general conditional process model was proposed for two-instance repeated measures designs with one moderator and one mediator. Simplifications of this general model correspond to more commonly used moderated mediation models, such as first and second-stage conditional process analysis
Two-condition within-participant statistical mediation analysis: A path-analytic framework	Montoya & Hayes, 2017	Use of path-analytic for models with multiple mediators, operating in parallel and serially. Guidelines for SPSS, SAS, and Mplus were provided to conduct these analyses
Block-wise model fit for structural equation models with experience sampling data	Norget & Mayer, 2022	A block-wise fit assessment was proposed for large models as an alternative. The entire model is estimated together, and block versions of common fit indices are then determined from smaller blocks of the variance-covariance matrix using simulated degrees of freedom
Parameter uncertainty in structural equation models: Confidence sets and fungible estimates	Pek & Wu, 2018	Two distinct types of parameter uncertainty were addressed, which are quantified by confidence sets (CSs) and fungible parameter estimates (FPEs). It was illustrated how CSs and FPEs provide unique information that leads to better scientific conclusions
Measurement equivalence: A comparison of methods based on confirmatory factor analysis and item response theory	Raju et al., 2002	A comparison was made between a linear method (confirmatory factor analysis) and a nonlinear method (item response theory and differential item functioning), focusing on their similarities and differences. Both approaches test the equality of true scores in two populations while holding the latent score constant, providing insights into measurement nonequivalence and its extent

(table 1 continues)

Table 1 (continued)

Title	Authors	Main findings
Melhores práticas para sua análise fatorial exploratória: tutorial no factor [Best practices for your exploratory factor analysis: A tutorial in factor]	Rogers, 2022	Exploratory factor analysis (EFA) is a widely used statistical method in management and social sciences. Its practice often relies on outdated heuristics, but it should be guided by solid theoretical principles, not just statistical fit, to ensure meaningful results
Métodos de correção de testes estatísticos em modelagem de equações estruturais [Methods for correcting statistical tests in structural equation modeling]	Silva et al., 2024	The study compared different statistical tests in SEM with increasing sample sizes violating normality assumptions. The H0SB, TM&E, and TY tests showed distinct behaviors, with H0SB supporting smaller samples (100 cases) and rejecting larger ones (≥ 500 cases), while TM&E and TY supported models with more cases (≥ 500)
Modelos latentes e slopes randômicos para análise de moderação e mediação [Latent models and random slopes for moderation and mediation analysis]	Valentini et al., 2018	Complex mediation and moderation models are crucial for understanding psychological phenomena. However, instrument structure and imprecision can affect variable relationships. Models that correct parameters with error estimates are essential. When slope variance is small, random slopes and latent interaction models are similar. Path analysis, while effective for moderation effects, underestimated direct and indirect mediation effects

Confirmatory Factor Analysis (CFA)

CFA is a multivariate model for analysis of structure of covariance in SEM. Through CFA, the measurement model can be contrasted according to the empirical data present in a given group, and this sample represents, albeit theoretically, what are the particularities of what is being studied (Dhaene & Rosseel, 2023).

Widely used in social science research, this statistical method describes the variability between observable variables correlated with a smaller number of unobservable variables. In CFA, factors are not directly observed and represent the cause of the relationship between unobserved variables (Silva et al., 2024).

This inferential tool makes it possible to perform validity hypothesis tests in a previously defined model, that is, there is a prior expectation of the relationship between the variables. The factors are estimated to provide explanations for covariances between the observed variables (Goretzko et al., 2024).

First-level CFA refers to the fact that one or more latent variables explain the observed variables. In second-level or hierarchical CFA, latent constructs explain the latent first-level variables. Thus, the correlations lose space to the saturations of the same factors, generating a new exogenous variable of a higher order, and in this variable the constructs of the first level are grouped. The CFA for the two-factor model is able to capture two distinct sources of variance: specific variance, originated in the factors themselves, and the common variance, shared by all items, known as the general factor or global mismatch. The two-factor model enables the discovery of new and latent structures (Rogers, 2022).

CFA goes through different stages: model specification, identification, parameter estimation, model adjustments, interpretation, and respecification. In the model specification, hypotheses are constructed of which indicator variables establish a relationship with which factors. This specification is carried out using the path diagram. For the estimation of parameters, the measurement model is identified, obtained from an already known information base. In the estimation, parameters are estimated to indicate the factorial structure of the tested model and resulting in a covariance matrix that fits the original covariance matrix (Hair et al., 2017).

Another concept in CFA is the estimators, which can be defined as test indices for the adequacy of the model, with the objective of reducing the differences or residuals between the sample covariance matrix, which contains empirical data, and the residual covariance matrix. The most commonly used methods are: maximum likelihood (ML), maximum likelihood robust (MLR), diagonally weighted least square (DWLS), weighted least square mean and variance parameter estimator (WLSMV) (Dhaene & Rosseel, 2023; Kelcey et al., 2020; Rogers, 2022; Silva et al., 2024).

ML performs estimates by likelihood with continuous data (Dhaene & Rosseel, 2023; Silva et al., 2024). The MLR is an estimation method based on restricted maximum likelihood, it is robust and not sensitive to the normality of the data (Kelcey et al., 2020). DWLS is an estimation method that considers the level of ordinal measurement of data and requires a larger sample of data. Finally, WLSMV is a robust weighted least squares estimator and adjustments are made as a function of mean and variance, allowing the categorical nature of the indicators to be considered. It is commonly used with categorical variables, known as polytomous variables, assuming the assumption of an underlying variable and requires a larger sample set. It is robust when there are deviations from normalities (Rogers, 2022).

Another important point in CFA is model fit ratios, which correspond to model fit indicators. There are several fit indices and the most used are: chi-square, comparative fit index (CFI), Tucker-Lewis index (TLI), root-mean-square error of approximation (RMSEA), and standardized root-mean-square residual (SRMR) (Silva et al., 2024).

The chi-square is an index for measuring the difference between the covariance matrix of the observed data and the covariance matrix used in the model. It is an adjustment test carried out according to the sample size. In this index, lower values generate higher levels of significance, that is, the lower the chi-square value, the better the fit of the model (Silva et al., 2024).

The CFI is a comparative adjustment index, which provides relative improvement in the fit of the researched model in relation to the standard model, it is generally independent, in which the covariances of all the observed variables are zero. In other words, it is an incremental fit index that allows the comparison of a given specified model with a model that is null, ensuring that models that have not been well specified are accepted. Thus, CFI values range from 0 to 1 and values higher than .95 determine the good fit of the model, accepting values above .90 (Norget & Mayer, 2022; Silva et al., 2024).

The TLI is a comparative adjustment index, but it compensates for the complexity of the model, including the penalization function from the inclusion of more estimated parameters, which may not be able to improve the fit of the model (Pek & Wu, 2018). TLI index values range from 0 to 1, and their interpretation is similar to the CFI interpretation (Norget & Mayer, 2022).

The RMSEA is a square root index of the mean of the approximation error that estimates how well the model parameters can reproduce population covariance. Therefore, it is a parsimonious correction index that incorporates penalties depending on the number of estimated parameters. It is an absolute fit index, responsible for measuring the average discrepancy of the specified model and the observed data. The cut-off point for this index is .06 to .07. And, the index values between .05 and .08 demonstrate a reasonable adjustment for this index. Values above .10, on the other hand, demonstrate a poor fit (Norget & Mayer, 2022; Pek & Wu, 2018; Silva et al., 2024).

Finally, the SRMR corresponds to a measure of global adjustment, responsible for evaluating the difference between the observed correlations and those estimated in the model. Known as a "misfit" measure, it makes it possible to quantify mean square differences in each bivariate empirical correlation with the implied counterpart. In this case, the best value is zero, as it indicates a perfect reproduction of the empirical correlation matrix. High SRMR values correspond to a worse fit of the model (Goretzko et al., 2024). This index is the square root of discrepancies of the sample covariance matrix and the model. It is standardized, ensuring its independence from the scale of measurements of the variables of the referred model. Interpretation values range from 0 to 1. In cases with good fits, it can be less than .05. SRMR values of .10 are accepted. But values above .10 demonstrate a poor fit (Goretzko et al., 2024).

Interpretation and respecification are steps of CFA. Through interpretation, it is examined how the variables are grouped. The factors are named, in order to demonstrate how these variables establish a relationship with these factors. It is necessary that the factors receive a name that clearly explains the set of variables that belong to this factor. On the other hand, in the respecification, it observes the presence of high residues, since these may be the reflection of items considered redundant, in which the variances of errors may be correlated with each other. It allows the adjustment of the model used in the measurement, based on the adequacy of estimating parameters (Hair et al., 2017).

In short, in CFA, the measurement model is estimated, providing reliability and feasibility for the model, so that, at another time, the structural model can be estimated. Thus, it is possible to demonstrate, in a sample, which variables correlate and forming a new dimension for the model. From a practical point of view, CFA is used in psychological tests, enabling psychological instruments to be developed and adapted (Dhaene & Rosseel, 2023).

Path Analysis

Path analysis is an extension of the linear multiple regression model, which decomposes the association of variables as a function of distinct direct and indirect effects, in order to understand causal relationships considering second-order constructs. To this end, the variables are computed by item scores, either from the mean or the sum (Devlieger & Rosseel, 2017; Douma & Shipley, 2022).

However, unlike linear multiple regression, in which the use of a dependent variable is allowed, in path analysis it is possible to represent, estimate, and test theoretical models in more complex models, where there is the presence of multiple dependent variables. With this, the analysis of a set of relationships is carried out simultaneously. In practice, path analysis allows the identification of concurrent predictors and the direct and indirect relationships between explanatory variables (Douma & Shipley, 2022).

Structural Equation Modeling

Psychology uses data analysis techniques and procedures to know and understand complex phenomena and their relationships. To this end, the application of multivariate techniques is required to analyze the data and their relationship with the phenomena being studied (Silva et al., 2024).

SEM translates into the combination of factor analysis and regression, and enables the testing of psychometric factor structures based on confirmatory factor analysis (Pek & Wu, 2018; Silva et al., 2024). That is, it allows the analysis of relationships between multiple variables at a given time, regardless of whether the variables are latent or observed. With this, it combines traditional methodologies for analyzing multivariate data applied in the testing of theoretical models in the areas of Organizational and Work Psychology and others (Devlieger & Rossel, 2017; Kelcey et al., 2020).

SEM is a technique in which the relationships of a set can be examined. The identification and analysis of the multiple relationships of dependence between the variables is carried out by means of the path diagram. Thus, multiple variables are measured and then their relationships are tested. The use of multiple linear equations is the means by which the SEM allows the inclusion of direct and indirect effects and latent variables, those variables that are not directly observed (Silva et al., 2024). The correlation between the independent variables and the dependent variables can be translated into diagrams. Its application is important because it is a robust technique, ensuring that it is not necessary to confirm hypotheses of normal distribution (Silva et al., 2024).

In addition, SEM measures the relationships between factors. It allows the testing of structural and measurement models and offers a complete analysis of the interrelationships in the model. Through the SEM, the structural and measurement models are examined, respectively. And, it promotes an approximate and efficient, reliable estimation for the various separate multiple regression equations estimated concomitantly (Montoya, 2024).

The method of estimating the measurement coefficient in the SEM is the partial least squares (PLS), which is simple and flexible in relation to the distribution of data and in relation to the sample size. PLS is a variance-based partial least squares method known as PLS-SEM (Hair et al., 2017). In the PLS, the theory and the indicators are analyzed simultaneously. It does not generate a general adequacy index, which makes the validity of the model a function of the examination of structural paths and values, and acts with sampling of a nonprobabilistic nature. And the analysis takes place in two stages: in the first, it seeks to validate the adequacy of the constructs as a function of the relationship between indicators and latent variables or constructs, allowing the evaluation of the reliability and validity of the variables or constructs. In the second part, it verifies whether an exogenous latent variable establishes a relationship with an endogenous latent variable (Hair et al., 2017). From a practical point of view, PLS is a method of parametric statistics, in which support is offered for exploratory and confirmatory research (Hair et al., 2017).

Mediation and Moderation

Mediation is a statistical method that tests indirect relationships. Known as an intervening variable or mechanism. In the SEM, the direct effects and those mediated by other factors that make up the causal network of outcomes are estimated in the list of interest of the research. Thus, mediation demonstrates the indirect effects on the analyzed model (Goldsmith et al., 2018; Montoya, 2024).

To put it another way, in mediation it is observed that an independent variable impacts a dependent variable indirectly, that is, it is an indirect effect that occurs when the effect of an independent variable (predictor) on the variable as dependent is transmitted through a mediator (Goldsmith et al., 2018). An independent variable is the precursor of a mediating variable, which is a predictor of a dependent variable. Therefore, in mediation, the degree of intermediation of a variable in a causal sequence of a predictor with a dependent variable is evaluated (Valentini et al., 2018).

The mediator known as the causal variable of the intervention interferes in the relationship between the intervention and the final result. Generally speaking, a mediating variable must answer “how” or “why” of the relationship between the predictor and the final outcome (Montoya, 2024). Figure 2 represents the mediation model.

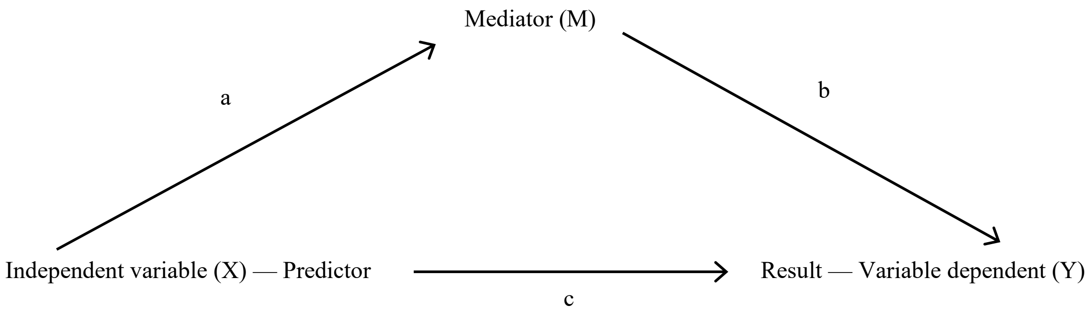

FIGURE 2
Mediation model

The mediation model returns the sum of direct, indirect, and empirical effects. Thus, the total effect (c) of an independent predictor variable (X) on the dependent variable (Y) is the sum of the direct effects X on Y and the indirect effect (Montoya, 2024).

Moderation occurs when the strength of the relationship between two variables depends on a third variable, the moderator. That is, it represents the capture of the direct effect of a given variable on the other, as a function of an individual difference (Montoya, 2024; Valentini et al., 2018).

The moderator variable transforms the relationship between the predictor and the dependent variable, which can increase the strength of the relationship between the predictor and the dependent variable or reduce the strength of the relationship (Montoya & Hayes, 2017). Figure 3 graphically represents the moderation model.

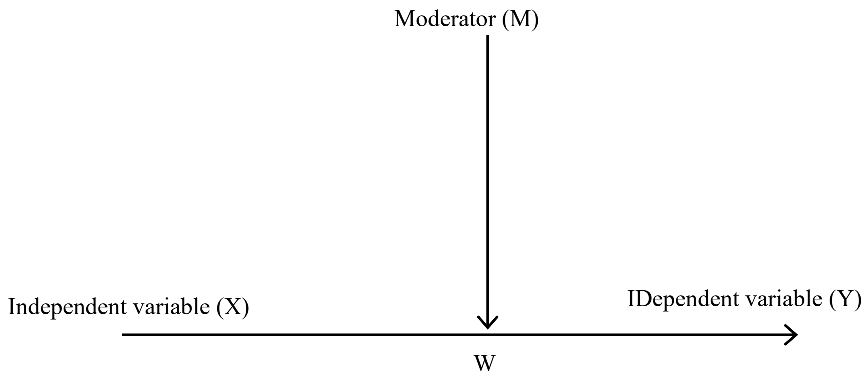

FIGURE 3
Moderation

The moderation model is obtained statistically, as a function of direct effects and interaction between variables. In this model, the moderator (M) interacts with an independent variable (X) in order to predict the dependent variable (Y). Thus, it demonstrates whether the regression of Y on the variable X has any variation as a function of W (Montoya, 2024; Valentini et al., 2018).

Statistical models of mediation and moderation, in practice, help in the identification of underlying mechanisms and conditions of a given model. Thus, they capture the complexity of social and psychological phenomena (Jacky et al., 2022).

The concomitant use of mediation and moderation results in mediated moderation or moderate mediation. Both are complex models of analysis, employed in studies and research, for evidence analysis or hypothesis testing of how certain mechanisms occur or how they help or hinder the effects (Valentini et al., 2018). They have equivalence and are used for modeling conditional processes, whose mediation and moderation processes are reconciled and form models of mediated moderation or moderate mediation analysis (Montoya, 2024).

In mediated moderation or conditional effect, there is an interactive effect between the independent variable and the moderator on another variable, the mediating variable. Its effects are observed when a given categorical or continuous variable affects the direction or intensity of the relationship between an independent variable and a mediator (Montoya, 2024; Montoya & Hayes, 2017; Valentini et al., 2018). Figure 4 shows a graphical representation of the mediated moderation model.

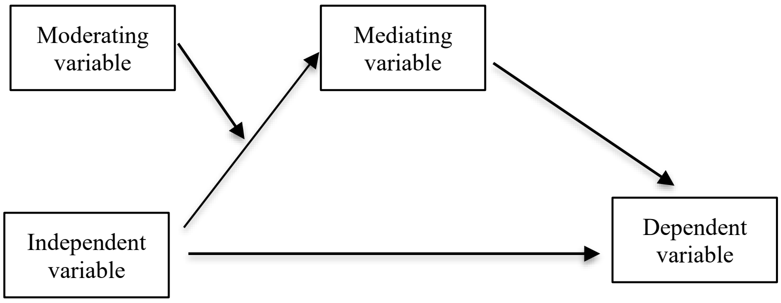

FIGURE 4
Mediated moderation

In the mediated moderation model, the interaction effect between independent variable X and moderator variable W on another variable Y that is dependent is observed. This interaction occurs by a mediator M. In mediated moderation, one of the paths A or B or both are moderated (Montoya & Hayes, 2017; Valentini et al., 2018).

In moderate mediation, there is a linear association between the indirect effect and the moderator (Jacky et al., 2022; Montoya, 2024). Moderate mediation postulates links or relationships between these variables, occurring when either path used or both paths are moderate (Valentini et al., 2018). Thus, the moderate mediation index corresponds to a test of equality between the conditional indirect effect intended for dichotomous moderators. The moderate mediation index is a mathematically formal test, used in the direct evaluation and quantification of the linear association between the indirect and moderating effect (Jacky et al., 2022). Figure 5 shows a graphical representation of a moderated mediation model.

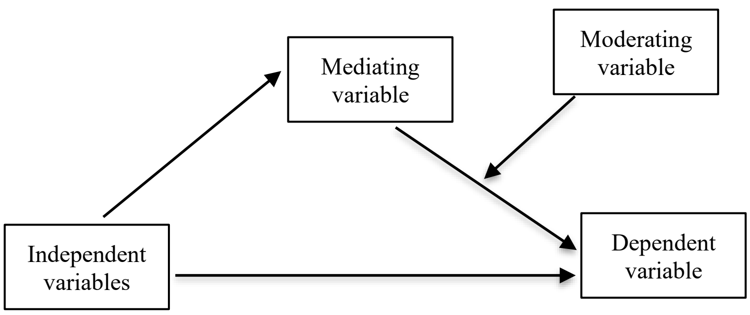

FIGURE 5
Moderate mediation

In the moderate mediation model, there is a moderation W of an indirect effect of an independent variable X on top of a dependent variable Y, and this occurs through a moderating variable M (Montoya, 2024; Montoya & Hayes, 2017). In practice, moderate mediation applies to the social and behavioral sciences demonstrating that the effect of interest refers to the size or direction of the effect of mediation conditional on moderators, as a result a moderate mediation effect is produced (Jacky et al., 2022; Valentini et al., 2018).

In short, the SEM is statistically robust because it allows the analysis of the relationships between latent and observed variables, configuring itself as a confirmatory statistical analysis, without data exploration. This method is well applied to observable and estimable relationships in which primary data make it possible to estimate variance and covariance (Pek & Wu, 2018). In practice, SEM allows the simultaneous estimation of a set of multiple regression equations, allowing the evaluation of the direct, indirect, and total effects of the variables on an outcome in which latent variables are included but not directly observed, providing the production of results that can be easily interpreted (Montoya, 2024; Valentini et al., 2018).

SEM, therefore, contributes to the development and theoretical advancement of psychology, enabling the resolution of empirical questions in this field (Kelcey et al., 2020). The improvement of measurement models favors psychology for the explanation of social and human phenomena, boosting the theoretical development of the area (Raju et al., 2002). With SEM, it is possible to carry out exploratory analyses, achieving statistical efficiency and depth, ensuring robustness and reliability of the data (Silva et al., 2024).

Suggested Future Studies

SEM as a statistical approach technique is little explored by researchers in the country. Thus, new studies are suggested for the construction of knowledge about the state of the art in SEM, allowing the technique to be better known and more used in order to improve causal analyses in health, especially in psychology and social sciences.

Studies are also suggested to deepen the step-by-step of the SEM in different models: direct relations, mediation and moderation, mediated moderation and moderate mediation, explaining the use of different software. Such studies may expand the list of studies on SEM for the testing of these models, as a statistical method for evaluating systematic and standardized variables ensuring accuracy of results.

Limitation

Although this study has offered a comprehensive overview of the state of the art of structural equation modeling (SEM) in psychology, some limitations must be considered. First, the systematic review was restricted to specific databases (Scielo, APA PsycInfo, and Pepsic) and may have limited the inclusion of relevant studies from other sources or nonindexed publications. In addition, the selection of descriptors (SEM, Psychometrics, and review reports) may have restricted the scope of the research, potentially excluding studies with different terminology for the application of SEM. Another limitation is the absence of empirical exploration. The study focused on the literature review without practical application or direct experimentation, preventing a critical analysis of the real application of SEM in different psychological contexts. Finally, the emerging nature of SEM in psychology research implies that the study's findings can quickly become outdated as new methods and applications of SEM emerge.

Final Considerations

The present study aimed to demonstrate the state of the art on SEM in the psychometrics scenario. With a multivariate statistical approach, this technique promotes the expansion of multiple linear regression analysis, analysis of variance, and confirmatory factorial analysis combining these analyses, which are important measurement procedures. Thus, it is possible to conclude that SEM is an important statistical method to analyze the relationships between variables, robust, reliable, and that it should be widely applied in health research, such as psychology for the technical analysis of research data.

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